

#1. Answer

$$163\frac{1}{3}$$

#2. Answer

8

#3. Answer

15

#4. Answer

$$-\frac{2}{3}$$

#5. Answer

2

#6. Answer

9

#7. Answer

2π

#8. Answer

31

#9. Answer

448

#10. Answer

e^3

#11. Answer

11

#12. Answer

7.5

#1. Solution:

$$\int_2^k [x^2 - 2x] dx = \left[\frac{x^3}{3} - x^2 \right]_2^k = \left(\frac{k^3}{3} - k^2 \right) - \left(\frac{8}{3} - 4 \right) \quad \text{Given } k = 9, \text{ we have } \Rightarrow \frac{729}{3} - 81 + \frac{4}{3} \Rightarrow 163 \frac{1}{3}$$

#2. Solution:

Since D and E are midpoints, \overline{DE} is congruent and parallel to \overline{FC} . The change in x coordinates and y coordinates between D and E is the same for points F and C . Therefore $3 - (-4) = a - (-2)$ and $k - 1 = b - (-3)$. So $a = 5$ and given $k = 7$, then $b = 3$. Hence, $5 + 3 = 8$.

#3. Solution:

On a number line, x must be between -12 and k . So the distance between -12 and k will give us an infinite number of solutions for t . Since $k = 3$, the distance between -12 and 3 is 15 , so $t = 15$.

#4. Solution:

Using Partial Fractions.
$$\frac{-2}{(n+1)(n+k)} = \frac{A}{n+1} + \frac{B}{n+k} = \frac{A(n+k)}{(n+1)(n+k)} + \frac{B(n+1)}{(n+1)(n+k)}$$

$$= \frac{An + kA + Bn + B}{(n+1)(n+k)} = \frac{(A+B)n + (kA+B)}{(n+1)(n+k)}$$

Given $k = 2$, we have $A + B = 0$ and $kA + B = 2$

$$A = -2 \text{ and } B = 2 \cdot \sum_2^{\infty} \left(\frac{-2}{n+1} + \frac{2}{n+2} \right) = \left(\frac{-2}{3} + \frac{2}{4} \right) + \left(\frac{-2}{4} + \frac{2}{5} \right) + \left(\frac{-2}{5} + \frac{2}{6} \right) + \dots$$

This is a telescoping series with sum of $-\frac{2}{3}$.

#5. Solution:

$P(x) = (x^2 + k)(x^4 - 16) = (x^2 + k)(x^2 - 4)(x^2 + 4)$. If $k > 0$, then there are 2 real zeros. If $k < 0$, then there are 4 real zeros. Since $k = 36$, there are 2 real zeros.

#6. Solution:

The derivative of $f(x) - f(2x)$ is $f'(x) - 2f'(2x)$. Substitute $x = 1$, we get $f'(1) - 2f'(2) = 5$ as row 1. Substitute $x = 2$, we get $f'(2) - 2f'(4) = k$ as row 2. The derivative of $f(x) - f(4x)$ is $f'(x) - 4f'(4x)$. So we need $f'(1) - 4f'(4)$. We obtain this from row 1 add two lots of row 2. $\Rightarrow (f'(1) - 2f'(2)) + 2(f'(2) - 2f'(4)) = 5 + 2 \cdot k$. Given $k = 2$, we have $5 + 2 \cdot 2 = 9$.

#7. Solution:

$r = \cos(\theta)$ is a circle with radius $\frac{1}{2}$ and center $\left(\frac{1}{2}, 0\right)$. $r = k \cos(\theta)$ is a circle with radius $\frac{k}{2}$ and center $\left(\frac{k}{2}, 0\right)$. The area between them is $\pi \cdot \left(\frac{k}{2}\right)^2 - \pi \cdot \left(\frac{1}{2}\right)^2$. Area = $\frac{\pi}{4}(k^2 - 1) = \frac{\pi}{4}(9 - 1) = \frac{8\pi}{4} = 2\pi$.

#8. Solution:

The 2 curves intersect at $(0, k)$ and $(1, 1+k)$. Area = $\int_0^1 ((x+k) - (x^2)) dx = \left[\frac{x^2}{2} + kx - \frac{x^3}{3} \right]_0^1$

$$= \left(\frac{1}{2} - \frac{1}{3} + k - 0 \right) = \frac{1}{6} + k$$

Given $k = 4$, $\frac{1}{6} + 4 = \frac{25}{6}$. So $a + b = 25 + 6 = 31$.

#9. Solution:

$(a + kb)^8 = a^8 + 8a^7(kb) + 28a^6(kb)^2 + 56a^5(kb)^3 + \dots$ So the fourth term is $56a^5(kb)^3$. Given $k = 2$, the coefficient of a^5 is $56 \cdot (2)^3 b^3 \Rightarrow 448b^3$.

#10. Solution:

$\lim_{x \rightarrow 1} [k \ln(x) + 1]^{k \ln(x)}$. Let $y = k \ln(x)$. We must find $\lim_{y \rightarrow 0} (y+1)^{\frac{3}{y}}$. Let $z = (y+1)^{\frac{3}{y}}$. By L'hopital's Rule, $\lim_{y \rightarrow 0} 3 \cdot \frac{1}{y+1} = 3 \ln(z) = 3$ or $z = e^3$.

#11. Solution:

$-3 < (3x - k) < 3 - 3 + k < 3x < 3 + k$. Given $k = 34$, we have $-3 + 34 < 3x < 3 + 34$. $31 < 3x < 37$.

$\frac{31}{3} < x < \frac{37}{3}$. The smallest integer is $\frac{33}{3} = 11$.

#12. Solution:

$$\begin{aligned} \vec{AB} &= \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}, \vec{AC} = \begin{bmatrix} 1 \\ n \\ 1 \end{bmatrix}, \vec{AB} \times \vec{AC} = \begin{bmatrix} 2 + 2n \\ -(3 + 2) \\ 3n - 2 \end{bmatrix} = \begin{bmatrix} 2 + 2n \\ -5 \\ 3n - 2 \end{bmatrix}. \text{ Since } n = 4, \text{ we have } \vec{AB} \times \vec{AC} = \begin{bmatrix} 10 \\ -5 \\ 10 \end{bmatrix}. \text{ Area} \\ &= \frac{1}{2} \cdot \left| \vec{AB} \times \vec{AC} \right| = \frac{1}{2} \cdot \sqrt{100 + 25 + 100} = \frac{1}{2} \cdot \sqrt{225} = 7.5 \end{aligned}$$
