#1. Answer
3

#2. Answer
7

#3. Answer
10

#4. Answer
2

#5. Answer
9

#6. Answer
3

#7. Answer
7

#8. Answer
5

#9. Answer
1

#10. Answer
2

#11. Answer
3

#12. Answer
3

#1. Solution:
Let $y = 2^x$. $y^2 = y + 6$ $y^2 - y - 6 = 0$ $(y-3)(y+2) = 0$ $y = 3$ $2^x = 3$ $x = \log_2(3)$ $N = 3$

#2. Solution:
$(2b+4)^2 = 6b^2 + 4b + 2$. $4b^2 + 16b + 16 = 6b^2 + 4b + 2$. $2b^2 - 12b - 14 = 0$ $b^2 - 6b - 7 = 0$. $(b-7)(b+1) = 0$. $b = 7$.

#3. Solution:
Notice that $25 = 10^{\log(25)}$. $25^{\frac{1}{\log(25)}} = \left(10^{\log(25)}\right)^{\frac{1}{\log(25)}} = 10^1 = 10$.

#4. Solution:
$f'(x) = e^x\left(x^2 + 2x\right)$. $f''(x) = e^x\left(x^2 + 4x + 2\right)$ a and b are the 2 roots of $x^2 + 4x + 2 = 0$. The product is $a \cdot b = 2$.

#5. Solution:
The divisors are: 1, 2, 4, 5, 10, 20, 25, 50, and 100.$The \prod uctis:$1*2*2*2*5*10*2*10*5*5*5*10*10*10.
=2^4*5^4*10^5=10^4*10^5=10^9.`

#6. Solution:
Let x be the fraction of girls and y the fraction of boys. $90x + 84y = 88$ and $x + y = 1$. $90(1-y) + 84y = 88$.
$90 - 90y + 84y = 88$. $6y = 2$ or $y = \frac{1}{3}$. $N = 3$

#7. Solution:
$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 2^4 \cdot 3^2 \cdot 2$. $\log(6!) = \log\left(2^4 \cdot 3^2 \cdot 5\right) = \log\left(2^4\right) + \log\left(3^2\right) + \log(5)$
$= 4\log(2) + 2\log(3) + \log(5)$. $a + b + c = 4 + 2 + 1 = 7$.

#8. Solution:
If $x > -3$, then $x^2 + 1 = 5x + 15$. $x^2 - 5x - 14 = 0$. $(x-7)(x+2) = 0$. $x = 7$ or -2. If $x < -3$, then
$x^2 + 1 = -5(x+3)$. $x^2 + 5x + 16 = 0$. No real roots. $7 - 2 = 5$.

#9. Solution:
Let $y = 4^x - 8$ and $z = 8^x - 4$. $y^2 + z^2 = (y+z)^2$. $y^2 + z^2 = y^2 + 2yz + z^2$. $y = 0$ or $z = 0$.
$4^x - 8 = 0$ or $8^x - 4 = 0$. $4^x = 8$ or $8^x = 4$. $2^{2x} = 2^3$ or $2^{3x} = 2^2$. $x = \frac{3}{2}$ or $x = \frac{2}{3}$. $\frac{3}{2} \cdot \frac{2}{3} = 1$.

#10. Solution:
$x^2 + x = 3x + 4$. $x^2 - 2x - 4 = 0$ $x = 1 \pm \sqrt{5}$ The points of intersection are
$(1+\sqrt{5}, 7+3\sqrt{5})$ and $(1-\sqrt{5}, 7-3\sqrt{5})$ The distance between these point is
$\sqrt{(2\sqrt{5})^2 + (6\sqrt{5})^2} = \sqrt{20+180} = 10\sqrt{2}$.

#11. Solution:
The terms are: $3, 7, 4, -3, -7, -4, 3, 7, \ldots$ The numbers repeat every 6 terms. 2023 divided by 6 has a remainder of 1, so $a_{2023} = a_1 = 3$.

#12. Solution:
By Fermat's Little Theorem, $2^{12} \equiv 1, \mod 13$. Since $100 = 12 \cdot 8 + 4'$ $2^{100} = \left(2^{12}\right)^8 \cdot 2^4$. The remainder is
$2^4 = 16 \equiv 3, \mod 13$.