

#1. Answer

9

#2. Answer

7

#3. Answer

3

#4. Answer

2

#5. Answer

36

#6. Answer

2

#7. Answer

3

#8. Answer

4

#9. Answer

2

#10. Answer

9

#11. Answer

34

#12. Answer

4

#1. Solution:

$$\Rightarrow \int_0^k (2x) dx = \left[x^2 \right]_0^k = k^2 - 0. \text{ Given } k = 3, \text{ we have } k^2 = 3^2 = 9.$$

#2. Solution:

$$\lim_{x \rightarrow 1} \frac{(2x+k)(\sqrt{x}-1)}{(2x-1)(x-1)} = \lim_{x \rightarrow 1} \frac{(2x+k)(\sqrt{x}-1)}{(2x-1)(\sqrt{x}+1)(\sqrt{x}-1)} = \lim_{x \rightarrow 1} \frac{2x+k}{(2x-1)(\sqrt{x}+1)} = \frac{2+k}{2} \text{ Given } k=7$$

then $\frac{2+7}{2} = \frac{9}{2}$. And so $9 - 2 = 7$.

#3. Solution:

The vertices of the triangle are $\pm \sqrt{\frac{j}{a}}$ and $(0, -j)$. To be equilateral, $2 \cdot \sqrt{\frac{j}{a}} = \sqrt{\frac{j}{a} + j^2}$ Since $k=10$ then

$$j=1 \Rightarrow \frac{2}{\sqrt{a}} = \sqrt{1 + \frac{1}{a}} \text{ So } \frac{4}{a} = 1 + \frac{1}{a} \Rightarrow a = 3.$$

#4. Solution:

Multiplying by 8 and 3 respectively, we have $8 \ln(x^2 y^3) = 8 \cdot 1 \Rightarrow \ln(x^{16} y^{24}) = 8$ and

$$3 \ln(x^5 y^7) = 3 \cdot k \Rightarrow \ln(x^{15} y^{21}) = 3k. \text{ And so we can get } \ln(xy^3) \text{ from}$$

$$\ln\left(\frac{x^{16} y^{24}}{x^{15} y^{21}}\right) = \ln(x^{16} y^{24}) - \ln(x^{15} y^{21}) = 8 - 3k. \text{ Given } k=2 \text{ we have } 8 - 6 = 2.$$

#5. Solution:

Using Heron's formula, we have $A = \sqrt{s(s-10)(s-17)(s-k)}$, where $s = \frac{10+17+k}{2} = \frac{27+k}{2}$. Since $k=9$, then

$$s = \frac{10+17+9}{2} = 18. \text{ And so } A = \sqrt{18 \cdot 8 \cdot 1 \cdot 9} = \sqrt{9 \cdot 2 \cdot 8 \cdot 9} = 3 \cdot 4 \cdot 3 = 36.$$

#6. Solution:

$5 \times 343 + 6 \times 49 + 7y + x = 512k + 7 \times 64 + 5 \times 8. \Rightarrow 2009 + 7y + x = 512k + 488. \Rightarrow 7y + x = 512k - 1521.$
 Since $k=3$, we have $7y + x = 15$. From the bases we know that $0 \leq x \leq 6$ and $0 \leq y \leq 6$, but it is easy to see $y=2$ and $x=1$.

#7. Solution:

Square both sides to get $j + x = x^2$. Since $k=7$, we have $j=6$. $6 + x = x^2. x^2 - x - 6 = 0 (x-3)(x+2) = 0 x=3$

#8. Solution:

$$f'(x) = \frac{(x^2+j)(1) - (x+1)(2x)}{(x^2+j)^2} = 0 \quad x^2 + j - 2x^2 - 2x = 0 \quad x^2 + 2x - j = 0 \quad x^2 + 2x - 3 = 0$$

$(x-3)(x+1) = 0. x=3$ and $x=-1$. The max is at $x=3$, so $a=3$. The min is at $x=-1$, so $b=-1$.

$$a-b = 3+1 = 4.$$

#9. Solution:

$$2(3x-2y)(3-2y') + 2(x-y)(1-y') = 0. 2[(3(k+2) - 2(k+3))(3-2y') + 2(k+2 - (k+3))(1-y')] = 0.$$

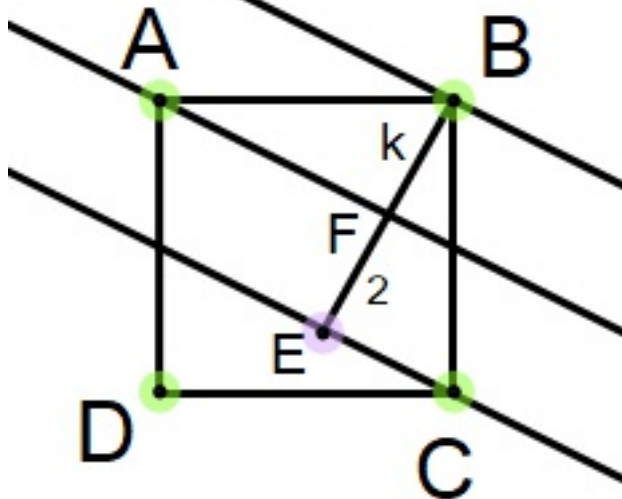
$$2(k)(3-2y') - 2(1-y') = 0. \text{ Given } k=1, 2(3-2y') - 2 + 2y' = 0. 6 - 4y' - 2 + 2y' = 0 y' = 2.$$

#10. Solution:

$$A = \int_0^{\frac{\pi}{2}} \frac{1}{2} r^2 d\theta = \frac{1}{2} \cdot j^2 \cdot \int_0^{\frac{\pi}{2}} \sin^2(2\theta) d\theta = \frac{j^2}{2} \cdot \int_0^{\frac{\pi}{2}} \frac{1}{2} (1 - \cos(4\theta)) d\theta = \frac{j^2}{4} \cdot \left[\theta - \frac{1}{4} \sin(4\theta) \right]_0^{\frac{\pi}{2}}$$

$$= \frac{j^2}{4} \left[\frac{\pi}{2} - \frac{1}{4} \sin(2\pi) \right] = \frac{j^2}{4} \cdot \frac{\pi}{2} \text{ Given } k=2, \text{ then } j=6. \Rightarrow \frac{9\pi}{2}.$$

#11. Solution:



$\triangle ABF \cong \triangle BCE$, so $EC = k$ and $EB = k + 2$. $(k + 2)^2 + k^2 = BC^2$ Given $k = 3$, we have $5^2 + 3^2 = BC^2 = 25 + 9 = 34$. The area is 34.

#12. Solution:

Let $A = \arctan\left(\frac{1}{2}\right)$, and $B = \arctan\left(\frac{1}{k}\right)$. So $\tan(A) = \frac{1}{2}$ and $\tan(B) = \frac{1}{k}$. $\tan(A + B) = \frac{\tan(A) + \tan(B)}{1 - \tan(A)\tan(B)}$
 $= \frac{\frac{1}{2} + \frac{1}{k}}{1 - \frac{1}{2} \cdot \frac{1}{k}}$ Given $k = 3$, we have $\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$. Since $\tan(A + B) = 1$, then $A + B = \frac{\pi}{4}$. So $N = 4$.