



Team Round 1

Mathematician

Time permitted: 45 minutes

Total questions in this Round: 12

Scratch paper, pen and electronic devices connected to the internet are essential

This is an open book experience

5 marks for each correct response, -2 for an incorrect response, -4 for an unanswered response

Give exact answers always

1. Find the general expression for A_n that satisfies the recurrence relation $A_n = 0.8A_{n-1} + 20 \quad \forall n \geq 1$ subject to the condition that $A_0 = 500$.
2. Anny, an aspiring mathematician, turned 25 today. She devised a thought experiment: if N is the sum of her current age raised to the power of the years 2025 and 2026 ie $N = 25^{2025} + 25^{2026}$. Find the sum of all those integers between 1 and 27 (inclusive) that is not a factor of N .
3. A ball starts rolling from the origin, along the positive x – axis with velocity $v = 0.5t^2 - 4t + 6$ cm/s, where $t \geq 0$. Determine the exact displacement of the particle from the origin, when it has moved a **total distance** of 18m.
4. Given that $z^6 - 6z^5 + 15z^4 - 20z^3 + 15z^2 - 6z = 0$, find the sum of the roots of $|z|$.
5. A normal pack of 52 playing cards is well-shuffled, so all outcomes are equally likely. Find the probability that the first two cards drawn include an ace and a heart.
6. The rule for divisibility of 3 in base 10 is: If the sum of the digits is divisible by 3, then the number is also. A tribe in S. America all have 6 fingers on each hand, so they work in base 36. The divisibility test for 3 in base 10 does not work in base 36 but instead works as a divisibility test for another integer. Find such integer n where $5 < n < 10$.

7. Let a, b , and c be single digit integers, i.e $0 \leq a, b, c \leq 9$, that satisfy $(a \times \overline{b9})^2 = \overline{5c756}$. Find $a + 2b + 3c$. (Note: $\overline{xy} = 10 \cdot x + y$, i.e it's a single number with x, y representing each digit, not multiplication. Analogous definition follows for more digits.)
8. A frog starts at $(1, 2)$ and jumps up, down, left, or right 1 unit at a time with the direction chosen uniformly random. The frog stops jumping when it reaches a side of the triangle formed by the points $(0, 0)$, $(4, 0)$ and $(0, 4)$. Find the probability that the frog's sequence ends on a coordinate axis.
9. If $3x^2 + 3y^2 = 10xy$, find the value of $\left| \frac{x+y}{x-y} \right|$.
10. ABC is a right triangle with $AB = 6$ and $AC = 4$. Let $D \neq A$ be a point on AC such that $\angle ABC = \angle DBC$ and $CD > CA$. Find the length of BD as a decimal.
11. Find the value of a such that $\sum_{n=0}^{\infty} e^{na} = 2$.
12. Given that $y = \frac{2}{x} + \cos(3x)$, there exists two expressions for $\frac{d^n y}{dx^n}$, one for when n is odd and one for when n is even. The general form is given below, solve for A, B and C. For odd n : $\frac{d^n y}{dx^n} = \frac{A}{x^{n+1}} + B \sin(3x)$. For even n : $\frac{d^n y}{dx^n} = \frac{A}{x^{n+1}} + C \cos(3x)$. Hint: B and C are trigonometric expressions.

End of Questions - now review your answers